




## Properties of Circles

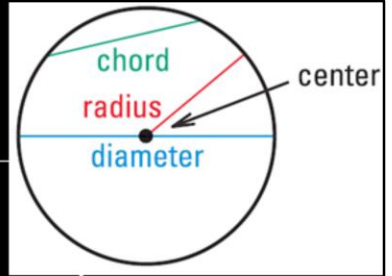
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Geometry  
Chapter 10

# Geometry 10

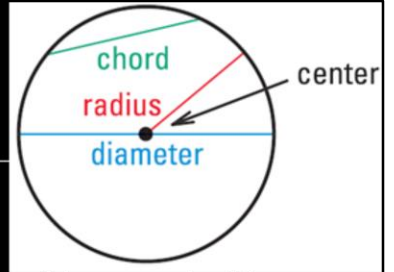
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- This Slideshow was developed to accompany the textbook
    - *Larson Geometry*
    - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
    - *2011 Holt McDougal*
  - Some examples and diagrams are taken from the textbook.
- Slides created by  
Richard Wright, Andrews Academy  
[rwright@andrews.edu](mailto:rwright@andrews.edu)

## 10.1 Use Properties of Tangents



- Circle
  - All the points a given distance from a central point in a plane
  - Named by the center
- Radius ( $r$ ) – the distance from the center of the circle to the edge.
- Chord – line segment that connects two points on a circle.

## 10.1 Use Properties of Tangents

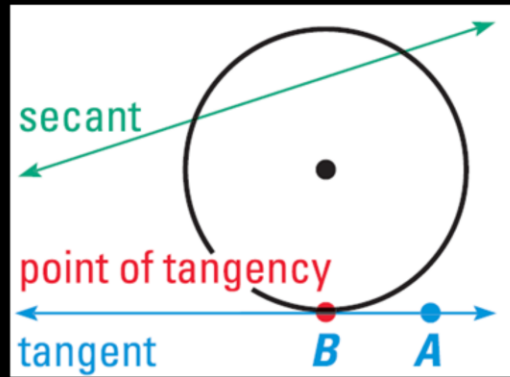


- Diameter (d) – chord that goes through the center of the circle (longest chord = 2 radii)
  - $d = 2r$
- What is the radius of a circle if the diameter is 16 feet?

→  $16/2 = 8$  feet

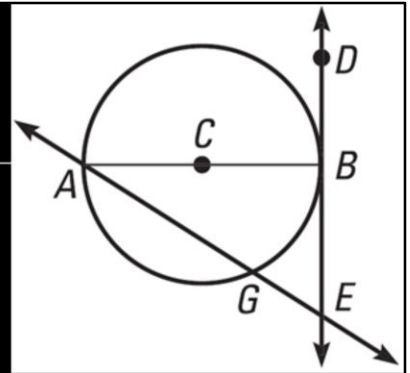
## 10.1 Use Properties of Tangents

- Secant
  - Line that intersects a circle twice
- Tangent
  - Line that intersects a circle once



## 10.1 Use Properties of Tangents

- What word best describes  $\overline{AG}$ ?
- What word best describes  $\overline{CB}$ ?
- Name a tangent and a secant.



Chord

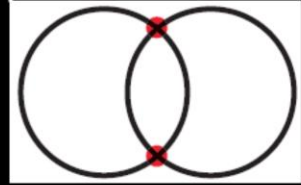
Radius

Tangent:  $\overleftrightarrow{DE}$

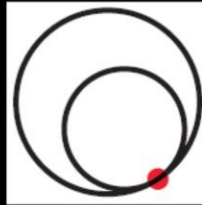
Secant:  $\overleftrightarrow{AG}$

## 10.1 Use Properties of Tangents

- Two circles can intersect in 2 points



- 1 point

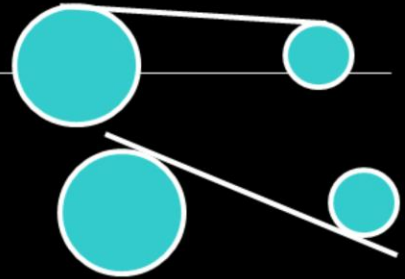


- No points

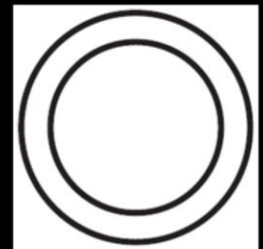
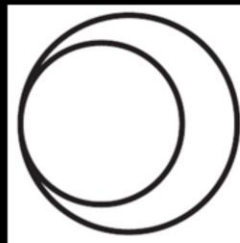
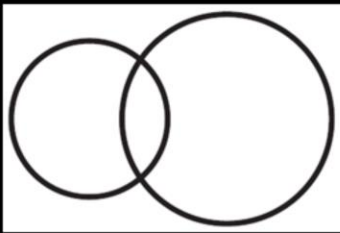


## 10.1 Use Properties of Tangents

- Common tangents
  - Lines tangent to 2 circles



- How many common tangents do the circles have?



2

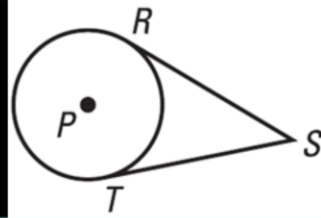
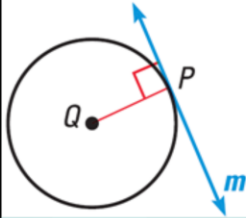
1

none



## 10.1 Use Properties of Tangents

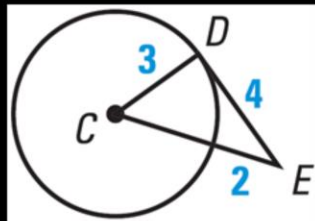
Tangent lines are perpendicular to radius.



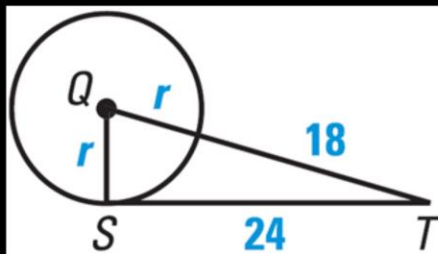
Tangent segments from the same point are congruent.

## 10.1 Use Properties of Tangents

- Is  $\overline{DE}$  tangent to  $\odot C$ ?

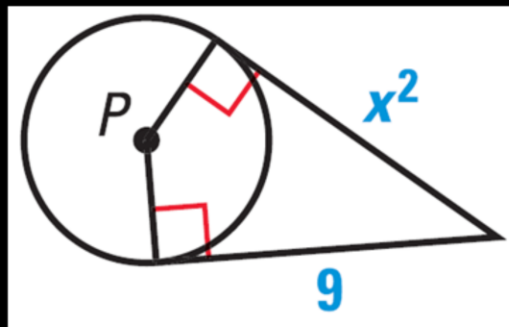


- $\overline{ST}$  is a tangent to  $\odot Q$ . Find the value of  $r$ .



## 10.1 Use Properties of Tangents

- Find the value of  $x$ .



- 655 #4-32 even, 36, 38, 43-47 all = 22



# Answers and Quiz

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- [10.1 Answers](#)
- [10.1 Homework Quiz](#)



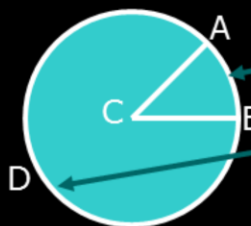
## 10.2 Find Arc Measures

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- How do you cut a pizza into eight equal pieces?
  - You cut in half, half, and half
- What measures are the angles in each piece?
  - $360 / 8 = 45^\circ$

## 10.2 Find Arc Measures

- There are  $360^\circ$  in a complete circle.
- Central Angle – Angle whose vertex is the center of the circle
- Arcs
  - An arc is a portion of a circle (curved line)
  - A central angle cuts a circle into two arcs
  - Minor arc – smaller of the two arcs – measures of arcs are the measures of the central angles
  - Major arc – bigger of the two arcs
  - Named  $\widehat{AB}$  or  $\widehat{ADB}$ 
    - use two endpoints to identify minor arc
    - use three letters to identify major arc





## 10.2 Find Arc Measures

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- Semicircle – arc if the central angle is  $180^\circ$
- Similar Circles – all circles are similar
- Congruent circles – same radius
- Congruent arcs – same radius and measure

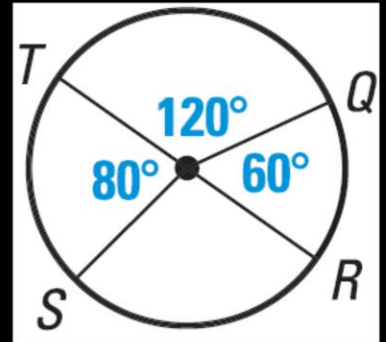
## 10.2 Find Arc Measures

- Identify as *major arc*, *minor arc*, or *semicircle*. Find the measure.

- $\widehat{TQ}$

- $\widehat{TQR}$

- $\widehat{QRT}$



$\widehat{TQ}$  minor arc;  $120^\circ$

$\widehat{TQR}$  semicircle;  $180^\circ$

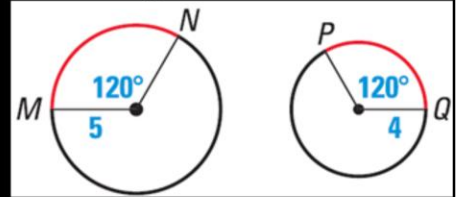
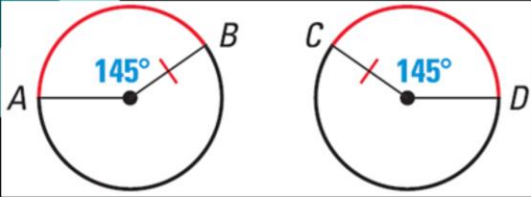
$\widehat{QRT}$  major arc;  $m\widehat{RS} = 360^\circ - 80^\circ - 120^\circ - 60^\circ = 100^\circ$

$$m\widehat{QRT} = 60^\circ + 100^\circ + 80^\circ = 240^\circ$$



## 10.2 Find Arc Measures

- Tell whether the red arcs are congruent.



- 661 #2-16 even, 20-24 even, 26-34 all = 20

Yes; same radius and angle

No; different radius



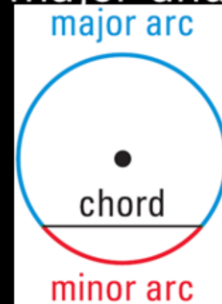
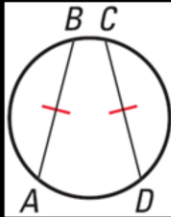
# Answers and Quiz

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- [10.2 Answers](#)
- [10.2 Homework Quiz](#)

## 10.3 Apply Properties of Chords

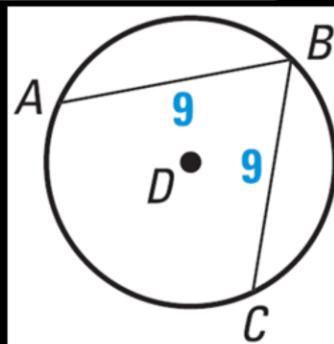
- Chords divide a circle into a major and minor arc.



In the same circle, or  $\cong$  circles, two minor arcs are  $\cong$  iff their chords are  $\cong$ .

## 10.3 Apply Properties of Chords

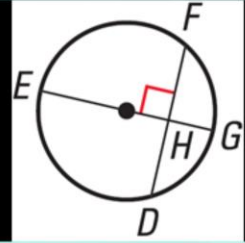
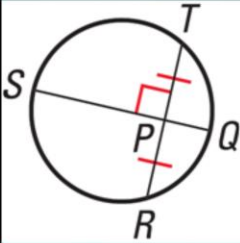
- If  $m\widehat{AB} = 110^\circ$ , find  $m\widehat{BC}$ .



$110^\circ$

## 10.3 Apply Properties of Chords

If one chord is  $\perp$  bisector of another chord, then the 1<sup>st</sup> chord is diameter.



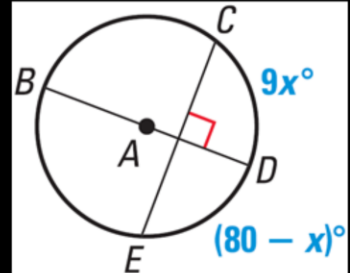
If a diameter is  $\perp$  to a chord, then it bisects the chord and its arc.

## 10.3 Apply Properties of Chords

- Find the measure of the indicated arc.

- $\widehat{CD}$

- $\widehat{CE}$



$$9x = 80 - x$$

$$10x = 80$$

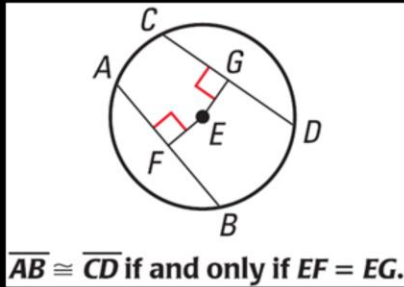
$$x = 8$$

$$m\widehat{CD} = 9(8) = 72^\circ$$

$$m\widehat{CE} = 2(72^\circ) = 144^\circ$$

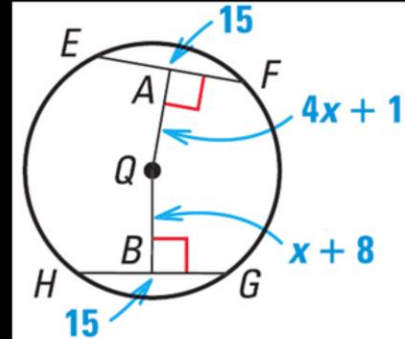
## 10.3 Apply Properties of Chords

In the same  $\odot$ , or  $\cong \odot$ , 2 chords are  $\cong$  iff they are equidistant from the center.



## 10.3 Apply Properties of Chords

- Find the value of  $x$ .



- 667 #4-20 even, 24, 30, 35-37 all = 14
- Extra Credit 670 #2, 4 = +2

$$\begin{aligned}4x + 1 &= x + 8 \\3x + 1 &= 8 \\3x &= 7 \\x &= \frac{7}{3}\end{aligned}$$





# Answers and Quiz

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- [10.3 Answers](#)
- [10.3 Homework Quiz](#)



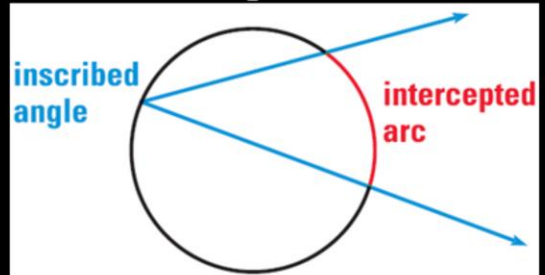
## 10.4 Use Inscribed Angles and Polygons

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- What does inscribed mean?
  - Writing ON something; engraving ON
- Inscribed angle means the vertex ON the circle.

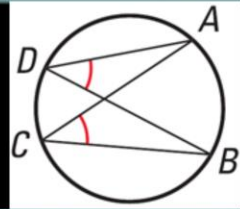
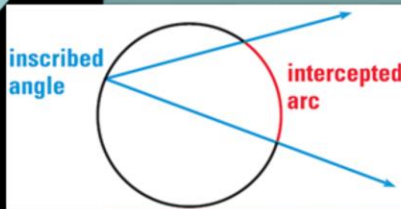
## 10.4 Use Inscribed Angles and Polygons

- Inscribed Angle
  - An angle whose vertex is on the edge of a circle and is inside the circle.
- Intercepted Arc
  - The arc of the circle that is in the angle.



## 10.4 Use Inscribed Angles and Polygons

The measure of an inscribed angle is  $\frac{1}{2}$  the measure of the intercepted arc.

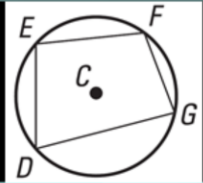
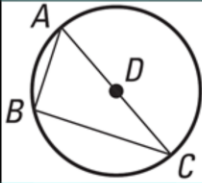


If two inscribed angles of the same or congruent circles intercept congruent arcs, then the angles are congruent.

## 10.4 Use Inscribed Angles and Polygons

If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle

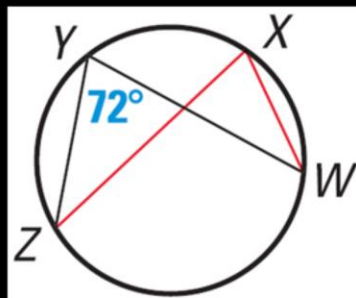
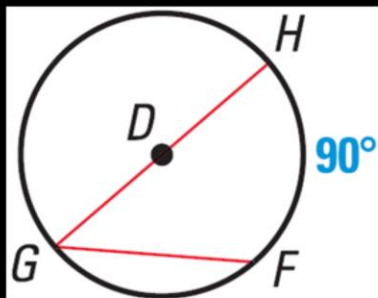
$$\frac{1}{2} 180 \text{ (semicircle)} = 90$$



If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

## 10.4 Use Inscribed Angles and Polygons

- Find the measure of the red arc or angle.

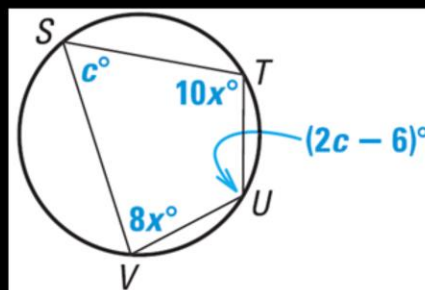
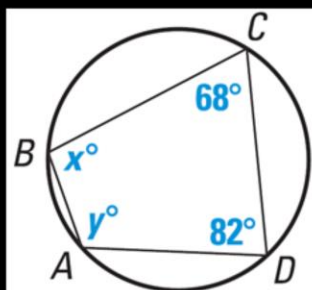


$$\frac{1}{2}90^\circ = 45^\circ$$

$$72^\circ$$

## 10.4 Use Inscribed Angles and Polygons

- Find the value of each variable.



- 676 #4-24 even, 28 36, 38, 40-46 all = 21

$$x + 82^\circ = 180^\circ$$

$$x = 98^\circ$$

$$y + 68^\circ = 180^\circ$$

$$y = 112^\circ$$

$$8x + 10x = 180$$

$$18x = 180$$

$$x = 10$$

$$c + (2c - 6) = 180$$

$$3c - 6 = 180$$

$$3c = 186$$

$$c = 62$$



## Answers and Quiz

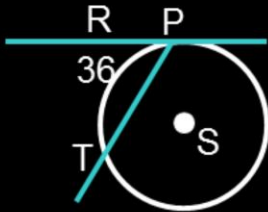
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- [10.4 Answers](#)
- [10.4 Homework Quiz](#)



## 10.5 Apply Other Angle Relationships in Circles

If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intercepted arc.



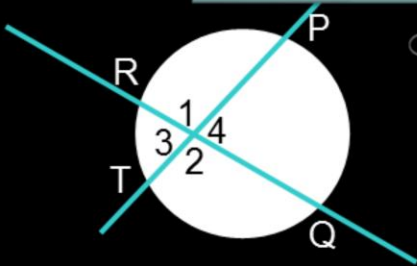
- Secant and Tangent intersect at point P on circle S. The angle formed measures  $36^\circ$ . What is the measure of the intercepted arc?

ANS:  $\angle = \frac{1}{2} \text{ arc} \rightarrow 36^\circ = \frac{1}{2} x \rightarrow x = 72^\circ$

## 10.5 Apply Other Angle Relationships in Circles

### Angles Inside the Circle Theorem

If two secants intersect in the interior of a circle, then the measure of an angle formed is  $\frac{1}{2}$  the sum of the measures of the arcs intercepted by the angle and its vertical angle.



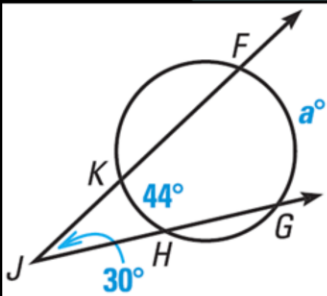
- $m\widehat{RT} = 50$ ,  $m\widehat{PQ} = 120$ . What is  $m\angle 3$ ?

$$\text{ANS: } m\angle 3 = \frac{1}{2} (m\widehat{RT} + m\widehat{PQ}) \rightarrow m\angle 3 = \frac{1}{2} (50 + 120) = \frac{1}{2} 170 = 85^\circ$$

## 10.5 Apply Other Angle Relationships in Circles

### Angles Outside the Circle Theorem

If two secants, tangents, or one of each intersect in the exterior of a circle, then the measure of the angle formed is  $\frac{1}{2}$  the difference of the measures of the intercepted arcs.



- What is the value of a?
- 683 #4-26 even, 32-39 all = 20
- Extra Credit 686 #2, 4 = +2

$$\begin{aligned}m\angle J &= \frac{1}{2}(m\widehat{FG} - m\widehat{KH}) \\30^\circ &= \frac{1}{2}(a^\circ - 44^\circ) \\60^\circ &= a^\circ - 44^\circ \\a &= 104\end{aligned}$$



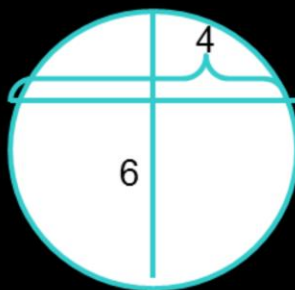
# Answers and Quiz

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- [10.5 Answers](#)
- [10.5 Homework Quiz](#)

## 10.6 Find Segment Lengths in Circles

- A person is stuck in a water pipe with unknown radius. He estimates that surface of the water makes a 4 ft chord near the top of the pipe and that the water is 6 ft deep. How much room is available for his head?

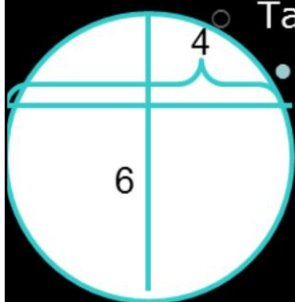


ANS: The chord can be subdivided into 2 ft and 2ft since the vertical line is a diameter. To answer the question we need to know the theorems in the section.

## 10.6 Find Segment Lengths in Circles

### Segments of Chords Theorem

If two chords intersect in a circle, then the products of the measures of the segments of the chords are equal.



- Take the example we started above.
- The segments of the horizontal chords are 2 and 2; the segments of the vertical chords are 6 and x

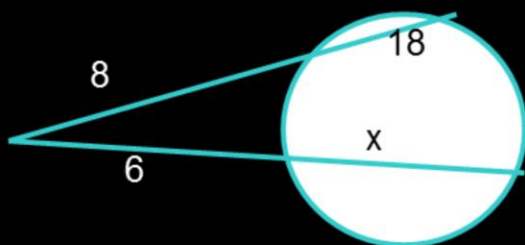
$$2(2) = 6x \rightarrow x = 4/6 = 2/3 \text{ ft Not much room for his head}$$

## 10.6 Find Segment Lengths in Circles

### Segments of Secants Theorem

If two secants are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant segment and its external secant segment.

○ Find  $x$  in the diagram.



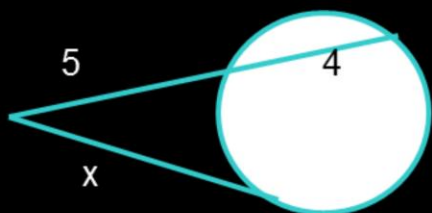
$$8(8 + 18) = 6(x + 6) \rightarrow 8(26) = 6x + 36 \rightarrow 208 = 6x + 36 \rightarrow 172 = 6x \rightarrow 28.67 = x$$

## 10.6 Find Segment Lengths in Circles

### Segments of Secants and Tangents Theorem

If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment is equal to the product of the measures of the secant segment and its external secant segment.

○ Find  $x$  in the diagram



$$x^2 = 5(4 + 5) \rightarrow x^2 = 5(9) \rightarrow x^2 = 45 \rightarrow x = 6.71$$





## 10.6 Find Segment Lengths in Circles

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- 692 #2-24 even, 30-42 even = 19



# Answers and Quiz

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- [10.6 Answers](#)
- [10.6 Homework Quiz](#)

## 10.7 Write and Graph Equations of Circles

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Standard equation of a circle

- $(x - h)^2 + (y - k)^2 = r^2$
- $(h, k)$  is the center of the circle and  $r$  is the radius

## 10.7 Write and Graph Equations of Circles

- Identify the center and radius of the given circles
  - $(x - 3)^2 + (y + 2)^2 = 16$
  - $x^2 + (y + 3)^2 = 4$
- Graph the above circles
  - To graph plot the center point. Then go up, down, left and right from the center the distance of the radius. You now have four points around the center. Connect the points with a circle.

→ center at (3, -2) and  $r = 4$

→ center at (0, -3) and  $r = 2$

## 10.7 Write and Graph Equations of Circles

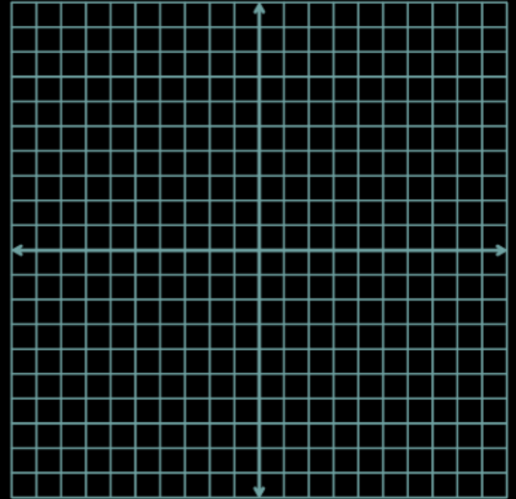
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- Write an equation for a circle with center  $(2, -4)$  and  $r = \sqrt{3}$

ANS:  $(x - 2)^2 + (y + 4)^2 = \sqrt{3}^2 \rightarrow (x - 2)^2 + (y + 4)^2 = 3$

## 10.7 Write and Graph Equations of Circles

- Graph  $(x - 4)^2 + (y + 2)^2 = 36$  and the line  $y = 2x - 2$  and state whether the line is a tangent or secant.
- 702 #2-38 even, 42, 46-54 even = 25
- Extra Credit 705 #2, 4 = +2



ANS: Graph the circle → center at (4, -2)  $r = 6$

Graph the line (use either slope intercept or table of values)  
It is a secant line



## Answers and Quiz

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- [10.7 Answers](#)
- [10.7 Homework Quiz](#)

# 10.Review

○  $712 \#1-19 = 19$

## 10 CHAPTER TEST

In  $\odot C$ ,  $B$  and  $D$  are points of tangency. Find the value of the variable.



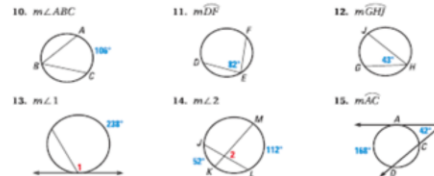
Tell whether the red arcs are congruent. Explain why or why not.



Determine whether  $\overline{AB}$  is a diameter of the circle. Explain your reasoning.



Find the indicated measure.



Find the value of  $x$ . Round decimal answers to the nearest tenth.



19. Find the center and radius of a circle that has the standard equation  $(x + 2)^2 + (y - 5)^2 = 169$ .